Thermal Conductivity Assessment in Limestone Rocks: Unveiling Patterns through P-Wave Velocity, Uniaxial Compressive Strength and Mineral Composition

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Abstract
Rock thermal conductivity is a critical property in the building and construction industry, playing a key role in optimizing energy efficiency. It guides material selection for insulation and ensures effective resistance to heat transfer within structures. This study introduces an alternative approach for estimating the thermal conductivity of rocks using an indirect method. The proposed approach leverages P-wave velocity, uniaxial compressive strength and mineral composition as predictive parameters.

This study examines the relationship between thermal conductivity and key rock properties, including P-wave velocity, uniaxial compressive strength and quartz content. A significant positive correlation was identified, highlighting the potential of these parameters as reliable predictors for estimating the thermal conductivity of rocks.

Keywords: Heat Transfer, Limestone, P-wave velocity, Thermal Conductivity.

Introduction
Exploring the thermal conductivity (TC) of rocks is crucial for advancing the construction and building sector, particularly in improving energy conservation. Moreover, TC is a fundamental parameter in geothermal engineering, civil and mining engineering[7,12,13] and heat flow studies[11]. Its importance has grown significantly in enhancing the understanding of low-temperature geothermal resources[6]. Energy preservation plays a crucial role in the formulation of every national energy strategy and its significance becomes even more pronounced in underdeveloped countries facing resource constraints[9]. Utilizing natural rock with minimal TC improves building insulation, providing an energy-efficient option.

Additionally, the significance of understanding the TC of rocks is escalating with the growing utilization of thermo-mechanical-hydrogeological computer models in rock mechanics studies. The determination of TC relies on measuring the temperature gradient within the rock and the applied heat[1]. It is crucial to account for to rectify heat losses. Precision in measuring heat flux is equally important. In rocks with low TC, achieving the required steady-state conditions is a time-consuming process.

Heat conduction follows Fourier's law as shown in equation 1:

$$Q = -kAt\frac{dT}{dx}, (J) \quad (1)$$

Where Q is the rate of heat transfer to the openings in time t (J), k is the thermal conductivity coefficient of the solid materials (W/mK), A is the surface area of the cross-sectional of a solid material (m^2), t is the time (s) and dT/dx is the temperature gradient (℃/m).

The thermal conductivity of rocks is typically assessed through three common methods: laboratory measurements, on-site measurements, or continuous *in situ* observations[4]. The construction industry frequently employs on-site measurements to rapidly ascertain ground properties, facilitating the design of ground loops for geothermal heating and cooling systems. An examination of prior studies indicates that the TC of rocks and concrete is predominantly influenced by factors such as rock type, porosity, P-wave velocity and moisture content[2,3,8,9,10,12]. The transport properties of fluid-rock interactions and the characterization of building materials are significantly influenced by porosity and TC.

Measuring P-wave velocity is a straightforward and uncomplicated process that can be conducted both in the field and laboratory settings. Its non-destructive nature and ease of application have led to an increasing utilization in geotechnical engineering. The intact rock properties are closely linked to the P-wave velocity of a rock, providing valuable insights into the structure and texture of the rock when measured in rock masses[5,14]. Uniaxial Compressive Strength (UCS) is a fundamental testing in geotechnical engineering, providing essential insights into the mechanical behaviour of rocks and their suitability for construction and insulation applications. UCS is closely tied to a rock's intact properties such as grain size, porosity and mineral composition which also influence TC. This study investigates how TC correlates with UCS, P-wave velocity and the mineral composition of rocks based on laboratory measurements.

Geology of the limestone deposits: Limestone rock samples were collected from the Cuddapah quarry. The Cuddapah basin is India's second-largest Proterozoic basin located in the eastern Peninsular region, covering 44,000 km^2 with a maximum width of 440 km. It is filled with 12

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km of varied carbonate-siliciclastic sediments and experiences minor igneous activity.

Material and Methods

Collection of rock samples: Limestone rock samples were collected from the Cuddapah quarry at different depths. Samples were gathered from five distinct locations in the Cuddapah region. To mitigate the impact of rock structure on thermal conductivity, we ensured the uniformity and isotropy of all selected samples for the research at a small scale, free from any noticeable bedding or cracks. Rock blocks were collected in the size of 20*10*10 cm size. For thermal conductivity assessment, disc samples were prepared using a coring and cutting machine, resulting in samples with a 50 mm diameter and a thickness of 20-25 mm. For P-wave velocity and UCS testing, core samples with a diameter of 54 mm and lengths ranging from 135 to 150 mm were made (Figure 1).

Measurement of thermal conductivity of rocks: This study employs the FOX50 steady-state apparatus to measure the TC of limestone rocks, utilizing its advanced capabilities to cover a range of 0.1 to 10 W/mK with an accuracy of approximately 3%. The system comprises of two circular plates with insulated cylinder guards, incorporating an upper fixed plate and a lower movable plate for placing samples. Employing an insulating cylinder guard effectively eliminates lateral heat, while a consistent air pressure of approximately 413kPa (60 psi) ensures optimal sample contact.

A disc sample is placed between the two round plates and is maintained at different temperatures. Eventually, a uniform one-dimensional temperature field is established within the sample's volume. With "WinTherm50" software, the TC of samples is measured and it takes 45 - 60 mins. The instrument is calibrated with the standard samples whose values are known before each test. The lab setup of FOX50 is shown in figure 2.

(a) (b)

Fig. 1: Rock sample photos for testing, (a) Disc sample for thermal conductivity test and (b) Cylindrical samples for P-wave velocity and UCS testing

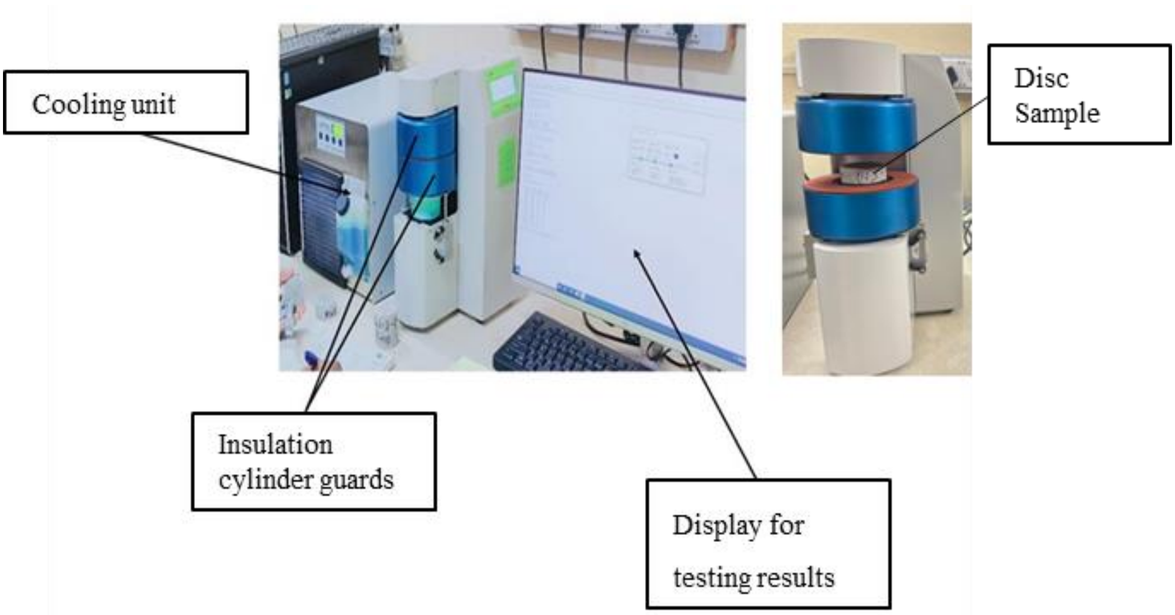

Fig. 2: Lab setup of FOX50

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P-wave velocity: The method of measuring P-wave velocity involves evaluating the speed of compression waves (P-waves) to analyse the density and elastic properties of a solid material. The quality of certain materials is linked to their elastic stiffness and evaluating this quality often involves measuring ultrasonic velocity and determining elastic properties through this method. Methods for P-wave velocity testing entail the measurement of the speed at which ultrasonic pulses traverse a material medium, utilizing an emitter, receiver and a device to indicate the time of travel. Figure 3 illustrates the measurement of P-wave velocity. Determining P-wave velocity involves measuring the time taken for a pulse to travel the distance between the transmitter and receiver.

The transducers are securely affixed to the specimen's surface and the display indicates the ultrasonic wave's travel time. Upon the stabilization of the measured value, a beep sound is emitted and the time and wave velocity are presented. The pulse velocity can be computed using the following formula (Equation 2):

$$V_p = \frac{L}{t} \text{ (m/s)} \quad (2)$$

where V_P is wave velocity (m/s), L is the length of the sample (m) and t is the transit time (s).

Uniaxial Compressive strength: Uniaxial Compressive Strength (UCS) is a key mechanical property influencing the thermal conductivity of rocks. UCS denotes a rock's ability to withstand compressive loads. To determine UCS, core samples with a diameter of 54mm and a length of 135-150mm are made. Prior to testing, the samples must undergo oven-drying.

The UCS is determined using compressive testing (Figure 4) where samples are positioned between two plates and subjected to a gradually applied load until failure. The recorded maximum load during the test is divided by the original cross-sectional area to calculate the UCS of the rock samples.

Fig. 3: Measuring the velocity of P-waves in rocks.

Fig. 4: Compressive testing machine

X-ray fluorescence (XRF) analysis: XRF (X-ray fluorescence) analysis of rocks entails evaluating the elemental makeup by detecting the distinct X-rays emitted when a sample is subjected to X-ray radiation. This method is widely used in geology to identify and quantify various elements in rocks, aiding in geological exploration and mineralogical research. The XRF technique was employed to determine the elemental composition of the rock samples. Prior to testing, the rock samples were segregated and crushed using a jaw crusher machine. Subsequently, the materials were finely ground into a powder of 75 µm using a micro ball mill device, as illustrated in figure 5.

Results and Discussion

Thermal conductivity of Cuddapah limestone rocks was measured and the values ranged from 2.60 to 3.12 W/mK, with an average of 2.93 W/mK. Additionally, the P-wave velocity measurements for the same region samples ranged from 5.3 to 6.6 km/s, with an average velocity of 6.2 km/s. The UCS values for the rock samples varied between 33.6 and 45.3 MPa, with an average UCS of 39.9 MPa (Table 1).

X-ray fluorescence (XRF) analysis was conducted on limestone samples, revealing major oxides such as SiO_2, Al_2O_3, Fe_2O_3, CaO, MgO, SO_3, MnO_2, TiO_2, K_2O, P_2O_5 and Na_2O (Table 2).

Fig. 5: Powder samples for XRF testing

Table 1
Thermal conductivity, P-wave velocity and UCS values of rock samples

Sample	Thermal conductivity (W/mK)		P-wave velocity (km/s)		UCS (MPa)	
	Range	Average	Range	Average	Range	Average
C1	2.75 – 3.10	2.93	5.8 – 6.5	6. 2	36.6 – 44.0	40.8
C2	2.77 – 3.10	3.00	5.9 – 6.6	6.3	37.3 – 45.3	41.9
C3	2.60 – 2.92	2.80	5.3 – 6.2	5.9	33.6 – 38.0	36.0
C4	2.97 – 3.10	3.01	6.3 – 6.5	6.4	40.0 – 43.0	41.1
C5	2.76 – 3.12	2.92	5.8 – 6.5	6.2	34.6 – 45.3	39.3
All samples	**2.6 – 3.12**	**2.93**	**5.3 – 6.6**	**6.2**	**33.6 – 45.3**	**39.9**

Table 2
XRF elemental composition of limestone rock samples

Major oxides (%)	Samples				
	C1	**C2**	**C3**	**C4**	**C5**
SiO_2	13.68	14.54	12.62	15.14	13.53
Al_2O_3	0.97	0.92	0.92	0.95	0.97
Fe_2O_3	0.48	0.52	0.47	0.53	0.49
CaO	47.33	47.67	47.43	47.53	47.49
MgO	0.89	0.75	0.82	0.76	0.88
SO_3	0.12	0.19	0.14	0.13	0.18
MnO_2	0.03	0.04	0.03	0.02	0.03
TiO_2	0.04	0.04	0.03	0.04	0.04
K_2O	0.25	0.21	0.24	0.21	0.19
P_2O_5	0.10	0.13	0.09	0.15	0.13
Na_2O	0.06	0.05	0.04	0.05	0.05
LOI	36.87	35.43	36.87	34.79	35.45

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Notably, CaO and SiO_2 emerged as primary oxides in the limestone. SiO_2, identified as the mineral quartz, boasting the highest TC among minerals (6-10 W/mK), significantly impacting the rock's overall thermal conductivity. It is noteworthy that SiO_2 is the second most abundant component in the limestone samples, influencing their thermal properties.

Predictive Modelling - Unveiling Trends with Regression Analysis: The investigation involved an analysis of P-wave velocity, UCS, Quartz (SiO_2) and TC results in rocks through the application of the regression method. This process enabled the derivation of the equation that best fits the data and the determination of the coefficient of determination (R^2) for each regression.

Figures 6a and 6b depict graphical representations illustrating the correlation between P-wave velocity and UCS with thermal conductivity. The results reveal a highly significant correlation between P-wave velocity and the TC of rocks, with an R^2 value of 0.89. Additionally, a correlation is observed between UCS and thermal conductivity, with an R^2 value of 0.85. Our findings indicate a positive linear correlation, suggesting that an increase in both P-wave velocity and UCS corresponds to an increase in the TC of the rock.

Quartz (SiO_2) demonstrates the highest thermal conductivity among all minerals, exerting a substantial impact on the overall TC of rocks owing to its prominent role in the mineral composition. Figure 7 illustrates a notable correlation between the percentage of quartz and the TC of rocks, indicating a positive correlation with an R^2 value of 0.84.

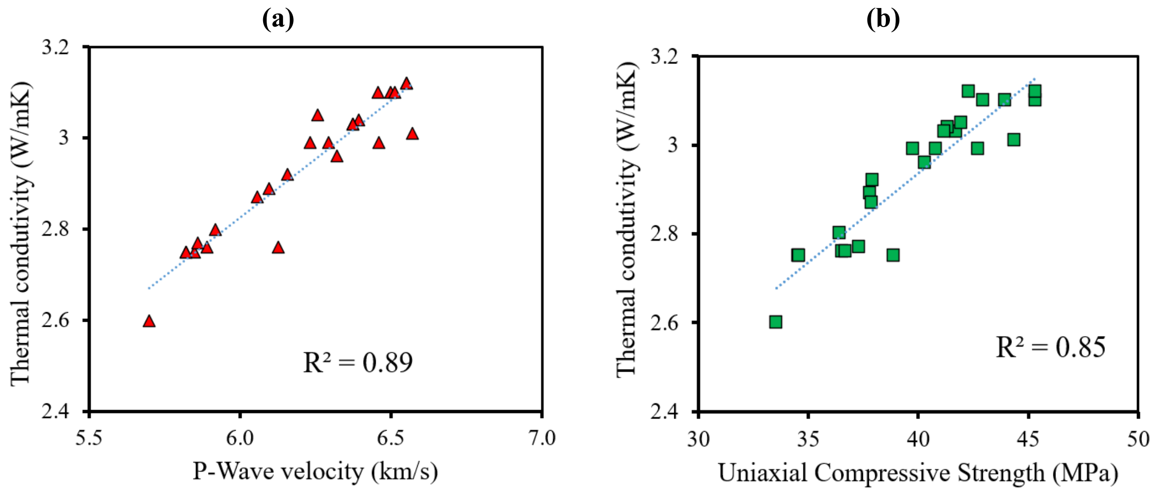

Fig. 6: Scatter plot of (a) TC versus P-wave velocity and (b) TC versus UCS

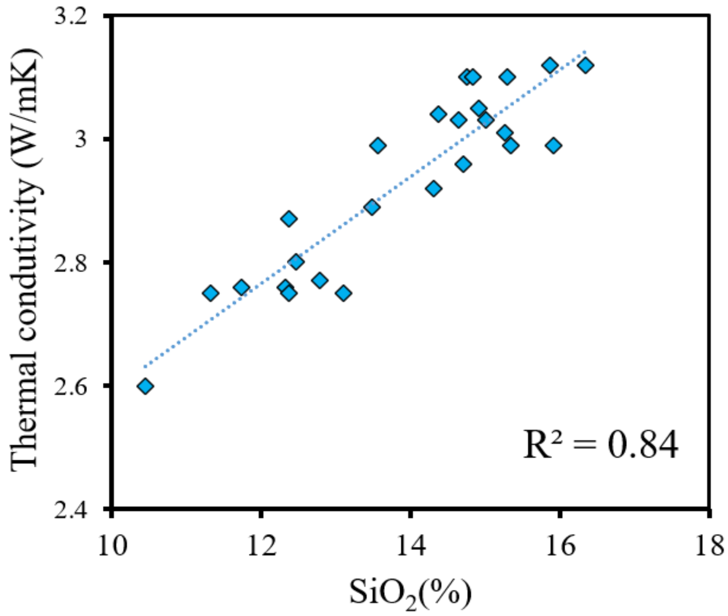

Fig. 7: TC versus SiO_2(%)

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Development of Regression Model for Prediction of the Thermal Conductivity of Rocks: A regression model has been formulated to predict the thermal conductivity of rocks is presented in equation 3:

$$TC = 0.619 + (0.2292 * V_p) + (0.01310 * \text{UCS}) + (0.0263 * Q) \quad (3)$$

where TC is the thermal conductivity of rocks, V_p is the P-wave velocity of rocks (km/s), UCS is the uniaxial compressive strength (MPa) and Q is the SiO_2(%).

The regression analysis details the relationship between performance and the influence of individual parameters which can be assessed through P-wave velocity, UCS and SiO_2. The model's coefficient of determination (R^2), illustrated in table 3, stands at 93.20%.

Table 4 indicates a statistically significant relationship for the independent variable with a P-value below 0.05. The model's effectiveness, as illustrated in table 3 with an R^2 value of 93.20%, signifies its statistical significance. Additionally, figure 8. displays the comparison between predicted and measured thermal conductivity, demonstrating an R^2 value of 0.93.

Conclusion

In this investigation, we assessed the thermal conductivity, P-wave velocity, UCS and mineral composition of limestone rock. In the mineral composition analysis using XRF, it is evident that quartz is the second-highest contributor in limestone rock samples and it has the highest thermal conductivity. The findings reveal that the TC of limestone rock can be accurately predicted using mathematical relationships involving P-wave velocity, UCS and the percentage of quartz. Through multivariate regression analysis, a robust correlation was observed between thermal conductivity and the variables of P-wave velocity, UCS and Quartz.

Notably, P-wave velocity consistently emerged as a significant factor in the developed regression models, underscoring its primary role in influencing the TC of rock samples. To enhance the scope of understanding, future studies should explore additional rock properties by incorporating a more comprehensive database.

Table 3
Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0416188	93.20%	92.23%	89.01%

Table 4
Regression analysis

Term	Coefficient	Standard Error Coefficient	T-Value	P-Value	VIF
Constant	0.619	0.328	1.89	0.073	
P-wave velocity	0.2292	0.0909	2.52	0.020	8.68
UCS	0.01310	0.00607	2.16	0.043	5.98
SiO_2	0.0263	0.0127	2.08	0.050	5.56

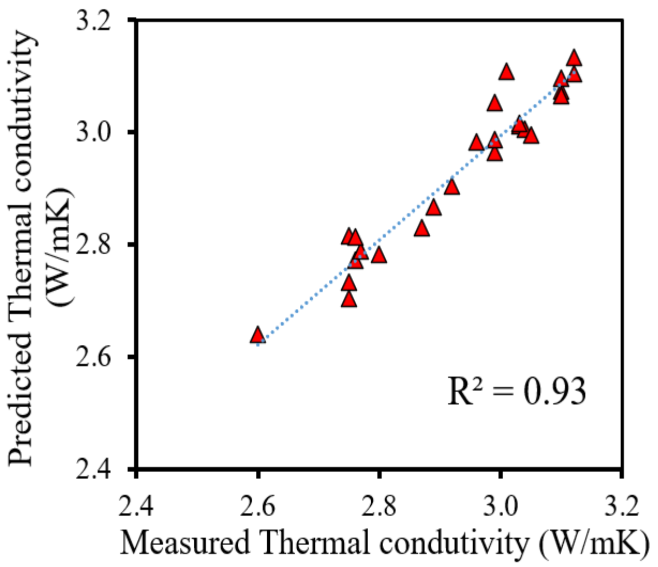

Fig. 8: Relationship between the predicted and measured TC of rock samples

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